MATH 141: Midterm 1

Name: _____

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
		90

1. If

$$f(x) = x^2 - x \qquad g(x) = 3x^2 - x + 1 \qquad h(x) = \sin(x) \qquad j(x) = 2^x$$

Evaluate, expand, and/or simplify the following:

(a) $h\left(\frac{\pi}{6}\right)$

(b) $j(1) \cdot h(0)$

(c) $f(x) \cdot g(x)$

(d) $f(x+h) - f(x)$

2. Short answer questions:

   (a) When you are given the directive ”Simplify this expression..” what does the word **simplify** mean??

   (b) True or false: We can simplify

$$\frac{3(x-2)^2(x+3)-4(x+2)(x-3)^2}{5x(x-3)^2(x-2)-4(x+3)}$$

   by crossing out the $x+3$.

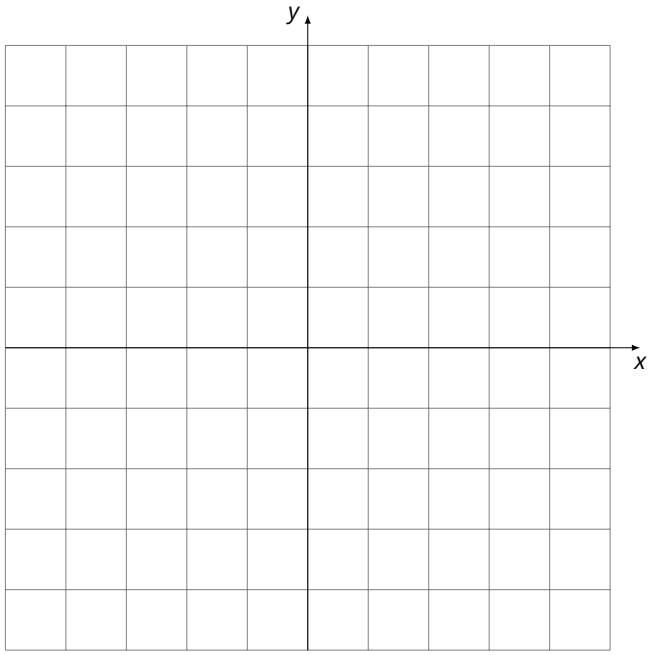
   (c) If a function is differentiable, is it continuous?

   (d) If $F(x)=\sin^3(x^2)$ find three functions f,g,h where $f\circ g\circ h=F$.

3. Suppose

$$f(x) = \begin{cases} x & x < 1 \\ -x^2 + 1 & x \geq 1 \end{cases}$$

(a) Sketch a graph of $f(x)$.



(b) What is $f(1)$?

(c) Does $\lim_{x \to 1} f(x)$ exist? If it does, find the limit. If not, explain why.

4. Perform the given instruction. Remember to use the relevant laws/properties and **fully simplify**.

   Not simplifying = lose points.

   (a) Expand and simplify: $\dfrac{3(x+h)^2 - 1 - (3x^2 - 1)}{h}$

   (b) Expand: $(x^3 + 6)(2x + 1) - (x^2 + x - 2)(3x^2)$

(c) Rationalize the numerator: $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

5. Draw the graph of a function which satisfies the following:

   (a) $f(0) = 1$

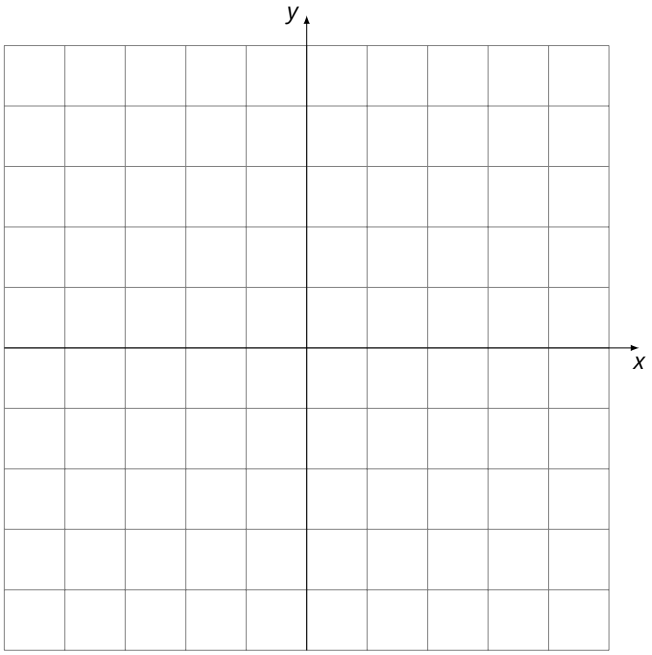
   (b) $f(2) = 1$

   (c) $\lim_{x \to 0} f(x) = 1$

   (d) $\lim_{x \to 2^-} f(x) = 0$

   (e) $\lim_{x \to 2^+} f(x) = 2$

   (f) $\lim_{x \to -2} f(x) = -\infty$



6. Consider this limit:

$$\lim_{h \to 0} \frac{\dfrac{1}{3+h} - \dfrac{1}{3}}{h}$$

(a) Try using Limit Laws to find the limit. What ends up happening?

(b) Now find the actual limit.

7. Use the **three-part definition of continuity** to prove the function

$$f(x) = \begin{cases} x(x-1) & x < 1 \\ 0 & x = 1 \\ \sqrt{x-1} & x > 1 \end{cases}$$

is continuous at the number $x = 1$.

Your answer needs to be as complete as my solution or points are lost.

8. Answer the following:

   (a) For a function $f(x)$, what is the **limit definition** of the derivative?

   (b) Suppose

   $$f(x) = 2x^2 - 1$$

   Using the limit definition of the derivative, find $f'(x)$.

   Not using the limit definition (i.e. using shortcuts) = 0 points.

9. Find the derivative of the following functions.

   (a) $f(x) = 534534532$

   (b) $g(t) = -t$

   (c) $f(x) = 4x^3 - 2x^2 + x - 5$

   (d) $g(\theta) = \theta \cdot \sqrt{\theta} \cdot \theta^3 \cdot \theta^4$